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ON THE DEPRESSION OF AN ALGEBRAIC EQUATION WHEN A PAIR OF ITS ROOTS ARE CONNECTED BY A GIVEN LINEAR RELATION.

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The usual method of depressing $f(x) = 0$, when a given linear relation exists between a and a' , two of its roots, is to substitute for x the value of a' (expressed in terms of a) ; the result, considered as an equation for a , has a common root with $f(a) = 0$, and a can then be found by the greatest common measure method.

Sometimes, however, the following method is to be preferred : Let a and a' be roots of the quadratic $X = x^2 - bx + c = 0$; divide $f(x)$ by X , leaving a remainder of the form $\varphi x + \theta$; to make this vanish identically, equate φ and θ separately to zero, express them as functions of a by the relation $b = a + a'$, $c = aa'$, and the given relation $la + ma' = p$; apply the G. C. M. method to find a the common root of $\varphi = 0$, $\theta = 0$.

In the former method both equations are of the same degree (n suppose), while in the latter φ is of the $(n-1)$ st degree in a and θ of the n th.

This will appear from the law of formation of the successive coefficients in the quotient and remainder arising from the division of $f(x)$ by $x^2 - bx + c$; it is evident from the process of division that each coefficient is equal to b times the preceding, minus c times the next preceding, plus the corresponding coefficient in the dividend, until we come to θ , in which case we omit the multiplier b . Now b , when expressed in terms of a , is of the first degree, and c is of the second ; hence the second, third, fourth, etc. terms of the quotient will involve a in the first, second, third, etc. degrees ; the last, or $(n-1)$ st term, will involve a in the $(n-2)$ nd degree, φ will be of the $(n-1)$ st degree in a , and θ of the n th. The induction in this case is so evident that it need not be formally given.

If the G. C. M. of φ and θ be of the second degree, it will give two values of a , and there are two corresponding values of a' , indicating that two pairs of the roots of $f(x) = 0$ are connected by the given relation ; similarly if $\varphi = 0$, and $\theta = 0$, have s common roots, there are s pairs of the roots of $f(x) = 0$ connected by the same relation.

Suppose, for example, that we have the equation

$$x^4 - 7x^3 + 8x^2 + 7x + 15 = 0, \quad (1)$$

and that it is known that two of its roots a and a' are connected by the relation $2a' - 3a = 1$.

To solve by the usual method, we substitute $\frac{1}{2}(3a + 1)$ for x in (1); thus

$$27a^4 - 90a^3 - 12a^2 + 82a + 105 = 0; \quad (2)$$

and then find the G. C. M. of (1) and (2).

By the second method

$$\begin{aligned}\varphi &= 65a^3 - 223a^2 + 59a + 75 = 0, \\ \theta &= 57a^4 - 167a^3 - 5a^2 + 19a - 120 = 0;\end{aligned}$$

and then apply the G. C. M. method as usual.

It may be noticed that the second method theoretically lessens the labor of obtaining the G. C. M., since φ is one degree lower than $f(a)$; practically, however, this is not always the case.

There is a case in which the usual method does not assist us in solving a proposed equation, namely, when we have an equation $f(x) = 0$, and it is known that the roots of this equation occur in pairs, and that *each* pair of roots a and a' satisfies the relation $a + a' = r$ (Todhunter, Theory of Equations, Art. 128).

The second method presents no such difficulty.

Suppose, for example, that we have the equation

$$f(x) = x^4 - 12x^3 + 13x^2 + 138x + 112 = 0,$$

where it is known that each pair of roots a and a' satisfies the relation $a + a' = 6$.

By the usual method we should proceed to investigate the common roots of the equations $f(x) = 0$ and $f(6 - x) = 0$. But these equations will be found to coincide completely, and do not afford a solution. The second method furnishes it easily. Write X in the form $x^2 - 6x + c$. $\varphi = 0$ vanishes, and $\theta = 0 = c^2 + 23c + 112$; whence by solving a quadratic equation we obtain c , and the roots are easily found. It may be remarked that when $f(x) = 0$ has equal roots, φ , expressed as a function of a , is the first derivative of $f(a)$.